NASA TECHNICAL NOTE



NASA TN D-3524

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION WASHINGTON, D. C.



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SUMMARY

A fixed-base simulator study was conducted to determine the ability of pilots to establish 80-nautical-mile (148.16-km) circular orbits about the moon by using a simplified guidance technique during retrothrust from a typical lunar approach trajectory. The pilot had control of thrust along the longitudinal axis and of vehicle attitude through an acceleration command system. No automatic damping or control were assumed. The general guidance procedure consisted of maintaining a constant angle between the vehicle thrust axis and the line of sight to the receding lunar horizon while applying thrust for a predetermined period of time. Several approach trajectories were considered for which the thrust angles and thrusting times were determined just prior to thrust initiation.

The results of the investigation showed that orbits near the desired parking orbit were established from the various approach trajectories considered when the instrumentation presented to the pilot consisted of only a three-axis gyro horizon nulled to the desired orientation in the plane of the approach trajectory.

When the pilot was required to track the lunar features in order to aline the space-craft with respect to the plane of the orbit and to apply thrust without the benefit of any instrumentation, his performance was degraded. The resulting orbits had altitudes between 70 nautical miles (129.64 km) at pericynthion and 100 nautical miles (185.20 km) at apocynthion for retrothrust from the nominal approach trajectory. When spacecraft attitude rate information was presented to the pilot, his performance in alining the spacecraft with respect to the plane of the approach trajectory was improved with a corresponding improvement in the established orbits. The resulting orbits had altitudes between 80 nautical miles (148.16 km) at pericynthion and 90 nautical miles (166.68 km) at apocynthion for retrothrust from the nominal approach trajectory.

INTRODUCTION

Spacecraft control during initial manned missions in selenocentric space will be provided, for the most part, by automatic guidance and control systems. However,

simplified guidance techniques that permit manual control of various phases of the lunar mission are of interest. These manual procedures can be used to monitor the performance of the automatic control modes, to provide backup control modes or, if sufficiently precise, might be considered as primary control modes. Such procedures should be very reliable and require a minimum of equipment.

In the analytical study of reference 1, the lunar horizon was used as a reference for thrust-vector orientation during the braking maneuver to establish near-circular orbits about the moon. The study indicated that, for a typical lunar approach trajectory, maintaining a constant angle between the thrust vector and the line of sight to the lunar horizon resulted in the efficient establishment of near-circular lunar orbits.

The present fixed-base simulator study was performed to determine, within the limits of this experiment, the ability of pilots to use the simplified guidance technique of reference 1 to establish near-circular orbits about the moon at an altitude of about 80 nautical miles (148.16 km). The present study employed a closed-circuit television system and permitted five degrees of freedom of the vehicle which was constrained to motion in the plane of the initial orbit. The equations of motion presented in the appendix were solved by using an analog computer operating in real time. The pilot closed the control loop and had direct input into the force and moment equations. In the investigation, thrust was initiated at an altitude of approximately 118.62 nautical miles (219.68 km) along a 70.5-hour-trip-time lunar approach trajectory.

SYMBOLS

Measurements for this investigation were made in U.S. Customary Units but are also given parenthetically in the International System of Units (SI). (See ref. 2.)

- F rocket thrust along X body axis, positive in direction of positive X axis, lbf (newtons)
- F_R, F_λ, F_β thrust components in spherical coordinates, referred to as radial, circumferential, and out-of-plane thrusts, respectively, lbf (newtons)
- ge acceleration at surface of earth due to gravitational attraction, 32.2 ft/sec² (9.814 meters/sec²)
- g acceleration due to gravitational attraction of the moon, ft/sec² (meters/sec²)
- h altitude above lunar surface, ft (meters) or nautical miles (kilometers)

I_{Sp} specific impulse, 313 sec

 I_X, I_Y, I_Z moments of inertia about X, Y, and Z body axes, respectively, $I_Y = I_Z$, slug-ft² (kilogram-meters²)

K angle between thrust vector and line of sight to lunar horizon (see fig. 7), radians or deg

MX,MY,MZ control moments exerted about X, Y, and Z body axes, respectively, ft-lbf (meter-newtons)

m vehicle mass, slugs (kilograms)

p,q,r vehicle angular velocities about X, Y, and Z body axes, respectively, radians/sec

 p_E, q_E, r_E components of vehicle angular velocity about X_E -, Y_E -, and Z_E -axes, respectively, radians/sec

R radial distance from center of moon, ft (meters) or nautical miles (kilometers)

t time, sec

 V_c characteristic velocity, $g_e I_{sp} \log_e \frac{m_i}{m}$, ft/sec (meters/sec)

 V_R, V_λ, V_β vehicle velocity components in spherical coordinates, referred to as radial, circumferential, and out-of-plane velocities, respectively, ft/sec (meters/sec)

W earth weight of vehicle, mge, lbf (newtons)

X,Y,Z vehicle body axes with origin located at vehicle instantaneous center of mass and with X-axis alined with vehicle axis of symmetry

 X_E, Y_E, Z_E reference axes with origin located at vehicle instantaneous center of mass and with X_E -axis alined with local vertical and positive outward, Y_E -axis positive eastward, and Z_E -axis positive northward (see fig. 1)

x,y,z assumed inertial reference axes with origin located at center of moon (see fig. 1)

normalized perturbation velocity component in xy-plane (see appendix)

 η normalized perturbation velocity component in out-of-plane direction (see appendix)

 ρ normalized perturbation displacement in radial direction (see appendix)

 $\delta_{\mathbf{F}}$ rocket throttle control displacement, radians or deg

 $\delta_X, \delta_Y, \delta_Z$ control displacements which produce control moments about ~X, ~Y, and ~Z body axes, respectively, radians or deg

 β angular displacement in out-of-plane direction, radians or deg (see fig. 1)

 λ angular displacement in xy-plane, radians or deg (see fig. 1)

 ψ, θ, φ Euler angles of rotation relating body axes and X_{E} -, Y_{E} -, and Z_{E} -axes (order of rotation θ, ψ, φ), radians or deg

 $\overline{\phi}$ angular orientation of vehicle defined as angle between XZ-plane and trajectory plane due to rotation about X-axis, referred to as roll angle, deg

 $\overline{\psi}$ angular orientation of vehicle defined as angle between XZ-plane and trajectory plane due to rotation about Z-axis, referred to as yaw angle, deg

$$\begin{pmatrix} l_1, l_2, l_3 \\ m_1, m_2, m_3 \\ n_1, n_2, n_3 \end{pmatrix}$$
 direction cosines

a,b,c,s quarternions or Euler parameters

Subscripts:

a conditions at apocynthion of established orbit

i initial conditions

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- o circular reference orbit conditions
- p conditions at pericynthion of established orbit
- ϵ error in conditions at burnout, $R_{\epsilon} = R_{actual} R_{perfectly flown}$

A dot over a symbol indicates differentiation with respect to time. A Δ preceding a symbol indicates the difference between the instantaneous value of that parameter and the reference orbit value, for example, $\Delta R = R - R_0$.

EQUATIONS OF MOTION

The equations of motion used in this study permitted five rigid-body degrees of freedom of the vehicle with the motion being constrained to the initial orbit plane. The force equations are written in spherical coordinates in perturbation form about a circular reference orbit which has an altitude equal to one-half the initial altitude. (See appendix.) The three moment equations were written with respect to the body axes. The assumed inertial reference frame was a fixed-axis system with its origin at the center of the moon (fig. 1), which was assumed to be a nonrotating homogeneous sphere. The pilot closed the control loop and had direct input into the force and moment equations. Vehicle mass and moments of inertia were varied as thrust was applied to account for mass reduction during thrusting. Mass changes due to moment control were neglected because they were small in comparison with the mass change due to thrust.

VEHICLE DESCRIPTION

The vehicle assumed in this study was an Apollo type of configuration in that a landing module was attached to the nose of the spacecraft (fig. 2). In this type of configuration, the pilot's view from the command position may be totally obscured by the landing module, and therefore necessitate the use of a telescope to view the lunar horizon. In the simulation, the assumed telescope had a field of view of approximately 30° about its look axis which was angularly offset 30° from the vehicle thrust axis.

In order to simplify the computer program, the vehicle used in the study was assumed to be a body of revolution. The vehicle had a single fixed engine which thrusted along the axis of symmetry. Thrust was either applied at the maximum level or was off. The maximum thrust level chosen for the study resulted in accelerating the vehicle at thrust initiation at $0.262g_e$ (F/W_i = 0.262). Upon orbit attainment, the acceleration

due to thrust was approximately $0.356g_e$ as a result of vehicle mass reduction. Specific impulse I_{SD} was assumed to be 313 seconds.

An acceleration command system which provided moment control about the vehicle body axes was assumed to be generated by reaction jets operating in pairs to produce pure torques. The variation of the vehicle moments of inertia with vehicle mass is presented in figure 3.

SIMULATOR

Cockpit and Controls

The general layout of the cockpit is presented in figure 4 which shows the relative position of the pilot's seat with respect to the television monitor and the instrument panel. It should be noted that of the numerous instruments included on this general-purpose panel, only the three-axis gyro horizon (eight ball) was used in this study. This instrument presented spacecraft attitude and attitude-rate information to the pilot.

Vehicle thrust was commanded by the use of an on-off controller located to the pilot's left. Attitude control about the three-body axes was commanded through an acceleration command system by using the three-axis attitude controller located on the pilot's right. Control inputs commanded by the pilot for attitude control were proportional to control deflection except for a small deadband around zero deflection (fig. 5).

Image Generation

The pilot's view through the simulated telescope was generated by a closed-circuit television system. The lunar model and television camera used in this study are shown in figure 6. The lunar model is a 20-foot-diameter (6.096 m) spherical fiber-glass shell. The model is back-lit and rotates about its polar axis to represent spacecraft translation over the lunar surface, that is, the model is driven by λ . The camera mount is positioned along the radial line from the center of the model to represent spacecraft altitude above the lunar surface. (The camera mount is driven by the expression for h.) The three rotational degrees of freedom of the simulated spacecraft were obtained by mounting the camera in a gimbal system which moved in response to the pilot's attitude control inputs. The limitations imposed upon the simulation by the equipment used are given in the following table:

Parameter	Limits		
Altitude	36 n. mi. (66.67 km) to 164.5 n. mi. (304.45 km)		
Pitch	-60° to +60°		
Yaw	-40° to +40°		
Roll	-40° to +40°		

In addition, a limited star field was produced through the use of 10 small lights located just above the simulated lunar horizon. (It should be noted that the star field was used for yaw reference only.) The resulting image was presented to the pilot on the television monitor mounted in the cockpit. A collimating lens mounted in front of the television monitor (see fig. 4) placed the image at infinity and thus added to the realism of the display. Included in the display were graduated crosshairs inscribed on the monitor face to provide spacecraft attitude reference. The reticle was graduated in 1^{0} increments in pitch and 4^{0} increments in yaw; however, pitch and yaw could be estimated to within about $1/2^{0}$.

LUNAR APPROACH TRAJECTORIES

Nominal Trajectory

The nominal lunar approach trajectory used in the present investigation (and in ref. 1) was a 70.5-hour-trip-time trajectory which had an altitude of 79.82 nautical miles (147.83 km) and a velocity of 8308.00 ft/sec (2532.28 m/sec) at pericynthion. The vehicle thrust level gave an initial deceleration of 0.262g_e. Thrust was applied when the altitude above the lunar surface reached 118.62 nautical miles (219.68 km).

Off-Nominal Trajectories

The off-nominal trajectories differed from the nominal trajectory at the 118.62-nautical-mile (219.68 km) thrust initiation altitude by various combinations of zero and ± 100 ft/sec (30.48 m/sec) errors in radial and circumferential velocity.

The following table lists the conditions at thrust initiation for the various trajectories flown during the study where the first set of conditions corresponds to the nominal trajectory:

Radial velocity, $ m V_R$		Circumferential velocity, V_{λ}		Altitude	
ft/sec	m/sec	ft/sec	m/sec	n. mi.	km
-1709.29	-520.99	8003.36	2439.42	118.62	219.68
-1709.29	-520.99	7903.36	2408.94		
-1709.29	-520.99	8103.36	2469.90		
-1809.29	-551.47	8003.36	2439.42		
-1809.29	-551.47	7903.36	2408.94		
-1809.29	-551.47	8103.36	2469.90		
-1609.29	-490.51	8003.36	2439.42		
-1609.29	-490.51	7903.36	2408.94		
-1609.29	-490.51	8103.36	2469.90	•	+

PILOTING PROCEDURE

The manual guidance procedure developed in reference 1 and used in the present study is designed to place the spacecraft in a nearly circular orbit about the moon at an altitude of approximately 80 nautical miles (148.16 km).

It was assumed that earth-based tracking was used to predict the value of the spacecraft velocity components upon arrival at the 118.62-nautical-mile (219.68 km) thrust initiation altitude and the time to go to thrust initiation. In the simulation, the pilot had 60 seconds between problem initiation and thrust initiation in which to determine the proper thrust angle and thrusting time to be used for the predicted velocity components of the particular approach trajectory. If nominal conditions prevail, the pilot directs thrust 38.6° above the line of sight to the receding lunar horizon for 315 seconds as determined through an iteration process in reference 1. After 315 seconds of thrusting, the spacecraft has descended to about 80 nautical miles (148.16 km) and thrust is terminated. A sketch of the trajectory relative to the lunar surface is presented in figure 7.

If off-nominal conditions exist, the pilot must modify the thrusting procedure in order to attain the desired circular orbit about the moon. In this case, the pilot enters the predicted radial and circumferential velocity components into the correction curves shown in figure 8 (from ref. 1) to obtain the proper thrust angle and thrusting time for the existing off-nominal approach trajectory.

The onboard equipment necessary to execute this piloting procedure consists of a telescope with graduated crosshairs, a stopwatch, and correction curves giving thrust angle and thrusting time as functions of the velocity components expected at 118.62 nautical miles (219.68 km). The correction curves, of course, could easily be omitted if the

thrust angle and thrusting time were determined on earth and relayed to the pilot. Alternately, an integrating accelerometer to determine velocity change would be equivalent to the timing method used in the simulation and would avoid errors due to changes in thrust level.

RESULTS AND DISCUSSION

Flights Using Three-Axis Gyro Horizon

In the first phase of the investigation, it was assumed that prior to the point of problem initiation the pilot had tracked the lunar surface as required to aline the space-craft thrust axis with the plane of the lunar approach trajectory and had set the null position of the three-axis gyro horizon (eight ball) with respect to that orientation. In this case the pilot used the telescope to maintain the proper thrust angle K and used the eight ball to keep vehicle yaw and roll angles zeroed.

Nominal trajectory.- The nominal trajectory was used primarily for pilot training and familiarization. The problem was initiated at an altitude of about 137.17 nautical miles (254.04 km) with the vehicle alined with the local horizontal. This orientation resulted in the thrust axis being directed about 29.2° above the line of sight to the lunar horizon. The pilot then had 60 seconds in which to pitch the vehicle to the desired attitude (K = 38.6°) before thrust was applied at an altitude of 118.62 nautical miles (219.68 km). The results of a typical piloted nominal trajectory are shown in figure 9. The time history shows that the pilot pitched to the proper thrust angle K = 38.6° approximately 50 seconds prior to the point of thrust initiation and generally maintained pitch attitude within about 1° of 38.6° for the 315-second thrusting period. Spacecraft roll and yaw angles generally varied within 2° about the desired 0° orientation. The pilot's performance for this flight was quite adequate as indicated by the errors at burnout of -3 ft/sec (-0.91 m/sec) in radial velocity, 7 ft/sec (2.13 m/sec) in circumferential velocity, and 1150 ft (350.52 m) in altitude.

A summary of the results of the nominal flights is presented in figure 10. These results (fig. 10(a)) show that, when the pilots terminated thrust, the error in radial velocity was generally less than 20 ft/sec (6.10 m/sec), the error in circumferential velocity was generally less than 10 ft/sec (3.05 m/sec), and the error in altitude was generally less than 5000 ft (1524 m). However, errors in radial and circumferential velocity as large as 37 ft/sec (11.28 m/sec) and 16 ft/sec (4.88 m/sec), respectively, occasionally occur. The resulting orbits, consequently, differ from the desired 80-nautical-mile (148.16 km) circular orbit. However, the established orbits have altitudes that lie between about 70 nautical miles (129.64 km) and 90 nautical miles (166.68 km) as shown in figure 10(b). The characteristic velocity required to perform the maneuver was

generally within 6.0 ft/sec (1.83 m/sec) of that required (3085 ft/sec (940.41 m/sec)) for a perfectly flown nominal trajectory.

Off-nominal trajectories.- A total of eight off-nominal trajectories were considered in the investigation. The flights were performed with the pilots using information given to them through the use of the correction curves shown in figure 8. The flight history of a typical off-nominal trajectory is presented in figure 11. The conditions at the 118.62-nautical-mile (219.68 km) thrust initiation altitude were a radial velocity of -1609.29 ft/sec (-490.51 m/sec) and a circumferential velocity of 8103.36 ft/sec (2469.90 m/sec). The time history shows that the pilot pitched to the proper thrust angle as indicated by the correction curves shown in figure 8, $K = 35.8^{\circ}$, approximately 40 seconds prior to thrust initiation and, in general, maintained that thrust angle within 1° for the 320-second thrusting period. The pilot maintained spacecraft roll and yaw angles within 2° of the desired 0° orientation. The pilot missed the burnout conditions of a perfect flight of the off-nominal trajectory by 7 ft/sec (2.13 m/sec) in radial velocity, 3 ft/sec (0.91 m/sec) in circumferential velocity, and 3920 ft (1194.82 m) in altitude.

Because there was no noticeable difference between the pilot's performance in flying the various off-nominal trajectories and the nominal trajectory, the resulting errors at burnout of both the nominal and off-nominal flights are presented together. The arithmetic mean and the standard deviation from the mean for the errors at burnout for both the nominal and off-nominal trajectories are presented in the following table for a total of 80 flights:

Parameter	Arithmetic mean		Standard	deviation
$egin{array}{c} {\sf V}_{{ m R},\epsilon} \ {\sf V}_{\lambda,\epsilon} \end{array}$	-7.25 ft/sec 7.6 ft/sec	(-2.19 m/sec) (2.32 m/sec)		(6.52 m/sec) (2.32 m/sec)
h_{ϵ}	-0.091 n. mi.		0.693 n. mi.	•

It should be noted that the errors in the burnout parameters are the difference between the actual values of the parameters and the values corresponding to a perfect flight of the particular trajectory.

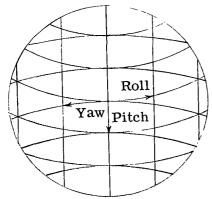
The altitudes at pericynthion and apocynthion of the resulting orbits are presented in figure 12. These results show that the established orbits lie between 60 nautical miles (111.12 km) and 105 nautical miles (194.46 km) above the lunar surface. In addition, approximately 80 percent of the orbits have altitudes that exceed 75 nautical miles (138.90 km) at pericynthion and are less than 95 nautical miles (175.94 km) at apocynthion. Thus, the pilot's use of the simplified guidance technique results in the establishment of safe orbits for the off-nominal trajectories considered. It should be noted that even for perfectly flown trajectories, the off-nominal conditions considered result in

orbits that lie between about 75 nautical miles (138.90 km) and 85 nautical miles (157.42 km) (see ref. 1). The characteristic velocity required for retrothrust for the off-nominal trajectories was generally within 120 ft/sec (36.58 m/sec) of that required for a perfectly flown nominal trajectory.

Flights Using No Instrumentation

In the second phase of the investigation, the pilot was required to aline the space-craft thrust axis with respect to the plane of the approach trajectory and pitch to the proper thrust angle without the benefit of instrumentation. The piloting procedure consisted of first orienting the spacecraft so that the lunar horizon is just in view and a maximum amount of the lunar surface is visible. The spacecraft is then rolled until the horizon appears level. The pilot then tracks the lunar features as they move with respect to the vertical crosshair and if necessary yaws the spacecraft until the landmarks move parallel to the vertical line. When the pilot is satisfied that he is alined with the plane of the approach trajectory, he pitches the nose of the spacecraft up and acquires a star. He then maintains zero yaw and roll angles by keeping the star at its acquired position relative to the vertical crosshair and by keeping the horizon level. The remaining task to be accomplished prior to thrust initiation is to pitch the spacecraft as required to locate the lunar horizon at the appropriate position corresponding to the proper thrust angle K.

Pilot control of spacecraft attitude is complicated to some extent by the fact that the look axis or center line of the simulated telescope is offset from the vehicle roll or thrust axis. This angular offset results in visual coupling between spacecraft roll and yaw motions as indicated in the following sketch:



The arrows shown in the sketch designate the directions of motion of a terrain feature for the indicated positive control inputs. For small displacements, it is difficult to differentiate between a positive roll motion and a negative yaw motion. Consequently, a number of flights were required to develop pilot proficiency. Little difficulty was encountered after the pilots had gained experience with the simulated telescope.

Because there was no noticeable difference between the pilot's performance in flying the various nominal and

off-nominal trajectories discussed previously, only the nominal trajectory was considered in this phase of the investigation. The spacecraft was initially offset $\pm 10^{\rm O}$ in roll and $\pm 10^{\rm O}$ in yaw from the desired orientation. The problem in this case was initiated at an altitude of approximately 164.5 nautical miles (304.65 km), and the pilot had 135 seconds,

between problem initiation and thrust initiation, to track the lunar landmarks, aline the spacecraft with respect to the plane of the approach trajectory, and pitch to the proper thrust angle.

The results of a typical piloted trajectory using no instrumentation are presented in figure 13. The spacecraft is initially rolled 10° and vawed -10° with respect to the desired orientation. After about 4 seconds, the pilot introduced a negative roll rate to level the horizon. The pilot attempted to remove the negative roll rate at about 18 seconds, at which time the roll angle was about -20. However, a small negative roll rate remained. The pilot then pitched down to track the lunar surface. At about 50 seconds. he began to correct the negative yaw angle. At about 80 seconds, the pilot believed he was on track and attempted to remove the positive yaw rate. The pilot overcorrected slightly, however, and introduced a slightly negative yaw rate. At about 95 seconds, the pilot pitched to about the proper thrust angle, $K = 38.6^{\circ}$, and continued to refine his pitch, roll, and yaw attitude until 134 seconds when thrust was applied. The pilot generally maintained pitch attitude within 10 of 38.60 during the 315-second thrusting phase. Spacecraft vaw and roll angles were generally kept within 40 of the desired 00 orientation but drifted between 00 and -40 during thrusting. (It should be noted that the negative bias is a function only of this particular flight, and roll and yaw in general varied about zero on a given run.) The attitude errors for this flight were somewhat greater than when the pilot was provided with a three-axis gyro horizon (fig. 9) and resulted in errors at burnout of -17 ft/sec (-5.18 m/sec) in radial velocity, 20 ft/sec (6.10 m/sec) in circumferential velocity, and -3750 ft (-1143.00 m) in altitude.

A total of 41 flights were performed without the aid of instrumentation. The arithmetic mean and the standard deviation from the mean for the errors at burnout and for the resulting pericynthion and apocynthion altitudes are presented in the following table:

Parameter	Arithmetic mean	Standard deviation	
${ m v}_{{ m R},\epsilon}$	-25.1 ft/sec (-7.65 m/sec)	31.0 ft/sec (9.45 m/sec)	
$v_{\lambda,\epsilon}$	19.0 ft/sec (5.79 m/sec)	7.2 ft/sec (2.19 m/sec)	
h_{ϵ}	-0.782 n. mi. (-1.45 km)	0.745 n. mi. (1.38 km)	
hp	75.573 n. mi. (139.96 km)	4.557 n. mi. (8.44 km)	
ha	95.097 n. mi. (176.12 km)	4.424 n. mi. (8.19 km)	

The standard deviation of the error at burnout in circumferential velocity for the flights made with no instrumentation and for the flights made using the three-axis gyro horizon are very nearly the same, about 7.5 ft/sec (2.29 m/sec). (See two preceding tables.) However, the arithmetic mean value of the error in circumferential velocity is 11.4 ft/sec (3.47 m/sec) greater in the absence of instrumentation. It is believed that this increase in the error in circumferential velocity at burnout is primarily due to the

increase in yaw angle error (which reduces the thrust component in the circumferential direction) that was experienced with the flights made without the benefit of instrumentation.

The primary factor contributing to the increase in the radial velocity error is believed to be the pilot's inability to concentrate on pitch attitude in the absence of instrumentation because of the drift rates present in yaw and roll attitudes. A 30 ft/sec (9.14 m/sec) increase in the error in radial velocity at burnout can be caused by a 0.5° increase in the average thrust angle error.

Although the resulting orbits differ from the desired 80-nautical-mile (148.16 km) circular orbit, the resulting pericynthion altitudes generally exceed 70 nautical miles (129.64 km) and the apocynthion altitudes are less than 100 nautical miles (185.20 km), thus, safe lunar orbits are established. The characteristic velocity required to perform the maneuver was generally within 6.0 ft/sec (1.83 m/sec) of that required for a perfectly flown nominal.

Flights Using Attitude Rate Information

Although without the benefit of onboard instrumentation the established orbits avoided impacting by a considerable margin, it was believed that the pilot's performance could be considerably improved if attitude rate information were included. In addition, it was believed that the inclusion of attitude rate information would not compromise the desired simplicity of the equipment. Rate information was presented to the pilot on the three-axis gyro horizon with the ball caged so that no attitude information was given. The piloting task was the same as for the case with no instrumentation. The pilot was required to track the lunar landmarks and aline the spacecraft with respect to the plane of the approach trajectory prior to thrust initiation. The spacecraft was initially offset from the desired orientation by various combinations of $\pm 10^{\circ}$ in roll and $\pm 10^{\circ}$ yaw.

The results of a typical piloted flight are presented in figure 14. The spacecraft is initially rolled $10^{\rm O}$ and yawed $-10^{\rm O}$ with respect to the desired orientation. The pilot first pitched the nose of the spacecraft down and then rolled the spacecraft to attain a nearly level horizon. He then tracked the lunar landmarks and at about 50 seconds reduced the negative yaw angle to about $-5^{\rm O}$. He continued the tracking process and reduced the yaw angle to $-3^{\rm O}$ at 70 seconds. He continued tracking and reduced the yaw angle to $0^{\rm O}$ at 90 seconds. He was then satisfied that he was on track and at 97 seconds pitched up and acquired a star. He then refined his thrust angle orientation and initiated thrust at 134 seconds. The pilot then maintained pitch attitude within about $1^{\rm O}$ of the desired $38.6^{\rm O}$ thrust angle while thrusting for 316 seconds, which is 1 second in excess of that desired. Spacecraft yaw and roll angles were generally maintained within $1^{\rm O}$ of the desired $0^{\rm O}$ orientation. The errors at burnout, 450 seconds, were 3 ft/sec

(0.91 m/sec) in radial velocity, 7 ft/sec (2.13 m/sec) in circumferential velocity, and 1550 ft (472.44 m) in altitude. The reduction in the errors at burnout are the result of reduced attitude errors achieved through the use of attitude rate information (compare figs. 13 and 14).

A total of 53 flights were performed in which the pilot was provided metered attitude rate information. The arithmetic mean and the standard deviation from the mean for errors at burnout and for the resulting pericynthion and apocynthion altitudes are presented in the following table:

Parameter	Arithmetic mean		Standard deviation	
${ m v}_{{ m R},\epsilon}$	4.7 ft/sec	(1.43 m/sec)	8.3 ft/sec	(2.53 m/sec)
$v_{\lambda,\epsilon}$	11.3 ft/sec	(3.44 m/sec)	3.0 ft/sec	(0.91 m/sec)
h_{ϵ}	0.14 n. m'i.	(0.25 km)	0.311 n. mi.	(0.58 km)
$h_{\mathbf{p}}$	80.041 n. mi.	(148.23 km)	0.513 n. mi.	(0.95 km)
ha	88.053 n. mi.	(163.07 km)	2.372 n. mi.	(4.39 km)

The reduction in burnout errors due to the inclusion of attitude rate information resulted in the establishment of orbits with altitudes at pericynthion that generally exceeded 80 nautical miles (148.16 km) and at apocynthion were generally less than 90 nautical miles (166.68 km). This range of altitudes corresponds to about a 10-nautical-mile (18.52 km) improvement in both pericynthion and apocynthion altitude over the orbits established without the benefit of any instrumentation. The characteristic velocity required to perform the maneuver was generally within about 6.0 ft/sec (1.83 m/sec) of that required for a perfectly flown nominal trajectory.

CONCLUDING REMARKS

A fixed-base simulator study has been conducted of the ability of pilots to establish close lunar orbits by using a simplified guidance technique during retrothrust from a typical lunar approach trajectory. The guidance technique required the pilot to maintain a constant angle between the vehicle thrust vector and the line of sight to the receding lunar horizon while thrusting for a predetermined period of time. At thrust termination, the vehicle should be in an approximately circular orbit at an altitude of about 80 nautical miles (148.16 km).

Within the limits of this simulation, the results of the investigation showed that, when the pilot was provided a three-axis gyro horizon nulled to the desired orientation in the plane of the approach trajectory, he could use the simplified guidance technique to establish orbits that were near the desired parking orbit from both the nominal and offnominal lunar approach trajectories. When the pilot was required to track the lunar

features in order to aline the spacecraft with respect to the plane of the approach trajectory and then apply retrothrust without the benefit of onboard instrumentation, the quality of the established orbits was degraded. However, the altitudes of the established orbits were generally between 70 nautical miles (129.64 km) at pericynthion and 100 nautical miles (185.20 km) at apocynthion for retrothrust from the nominal approach trajectory. The inclusion of spacecraft attitude rate information improved the pilot's performance and resulted in an improvement in the established orbits with altitudes generally lying between 80 nautical miles (148.16 km) at pericynthion and 90 nautical miles (166.68 km) at apocynthion.

The orbits established by this simple visual technique vary somewhat from the desired 80-nautical-mile (148.16 km) circular orbit; however, it is believed that subsequent circularization toward the desired altitude may be accomplished through the use of navigational techniques using spacecraft optics or earth-based tracking.

Langley Research Center,

National Aeronautics and Space Administration, Langley Station, Hampton, Va., March 2, 1966.

EQUATIONS OF MOTION

The equations of motion used in this investigation were formulated for analog computation by Roland L. Bowles of the NASA Langley Research Center. The translational equations of motion governing the behavior of a vehicle with respect to a nonrotating homogeneous spherical moon may be written in spherical coordinates as follows (see fig. 1):

$$\dot{V}_{R} = \frac{V_{\beta}^{2}}{R} + \frac{V_{\lambda}^{2}}{R} - \frac{g_{O}R_{O}^{2}}{R^{2}} + \frac{F_{R}}{m}$$
 (1)

$$\dot{\mathbf{V}}_{\beta} = -\frac{\mathbf{V}_{\mathbf{R}}\mathbf{V}_{\beta}}{\mathbf{R}} - \frac{\beta \mathbf{V}_{\lambda}^{2}}{\mathbf{R}} + \frac{\mathbf{F}_{\beta}}{\mathbf{m}}$$
 (2)

$$\dot{\mathbf{V}}_{\lambda} = -\frac{\mathbf{V}_{\mathbf{R}}\mathbf{V}_{\lambda}}{\mathbf{R}} + \frac{\beta\mathbf{V}_{\beta}\mathbf{V}_{\lambda}}{\mathbf{R}} + \frac{\mathbf{F}_{\lambda}}{\mathbf{m}}$$
 (3)

where β is assumed to be a small angle ($\sin \beta \cong \beta$ and $\cos \beta \cong 1$).

By choosing a circular reference orbit with an altitude of $h_0 = \frac{h_i}{2}$ and defining

$$\Delta \beta = \beta - \beta_{O} \tag{4}$$

where

$$\beta_{\mathcal{O}} = 0 \tag{5}$$

and

$$\Delta R = R - R_0 \tag{6}$$

$$\Delta V_{\lambda} = V_{\lambda} - V_{\lambda,0} \tag{7}$$

$$\Delta V_{\beta} = V_{\beta} \tag{8}$$

where

$$V_{\lambda,O} = \sqrt{R_{O}g_{O}}$$
 (9)

the following parameters may be formed:

$$\rho = \frac{\Delta R}{R_0} \tag{10}$$

$$\zeta = \frac{\Delta V_{\lambda}}{\sqrt{R_{O}g_{O}}} \tag{11}$$

$$\eta = \frac{\Delta V_{\beta}}{\sqrt{R_{O}g_{O}}} \tag{12}$$

The following three perturbation equations describing the translational motion of the vehicle may be obtained by substituting equations (10), (11), and (12) into equations (1), (2), and (3):

$$\frac{R_{O}}{g_{O}} \ddot{\rho} = \frac{\zeta(2+\zeta)+\eta^{2}}{(1+\rho)} + \frac{\rho}{(1+\rho)^{2}} + \frac{F_{R}}{g_{O}m}$$
 (13)

$$\sqrt{\frac{R_O}{g_O}} \dot{\eta} = -\sqrt{\frac{R_O}{g_O}} \frac{\dot{\rho}\eta}{(1+\rho)} - \frac{\Delta\beta(1+\zeta)^2}{(1+\rho)} + \frac{F_\beta}{g_O m}$$
(14)

$$\sqrt{\frac{R_o}{g_o}} \dot{\zeta} = \frac{(1+\zeta)}{(1+\rho)} \left(-\sqrt{\frac{R_o}{g_o}} \dot{\rho} + \Delta \beta \eta \right) + \frac{F_{\lambda}}{g_o m}$$
 (15)

Equations (14) and (15) may be integrated to obtain

$$\eta = \eta_{i} \left[\frac{1 + \rho_{i}}{(1 + \rho)} \right] - \frac{1}{\sqrt{\frac{R_{o}}{g_{o}}} (1 + \rho)} \int_{0}^{t} \left[(1 + \zeta)^{2} \Delta \beta + \frac{(1 + \rho)}{g_{o}^{m}} F_{\beta} \right] dt$$
 (16)

$$\zeta = \frac{\left(1 + \rho_{i}\right)\zeta_{i} + \rho_{i} - \rho}{\left(1 + \rho\right)} + \frac{1}{\sqrt{\frac{R_{o}}{g_{o}}}(1 + \rho)} \int_{0}^{t} \left[(1 + \zeta)\eta \ \Delta\beta + \frac{(1 + \rho)}{g_{o}^{m}} \ F_{\lambda} \right] dt$$
 (17)

If motion is now constrained to the initial orbit plane ($\Delta \beta = \eta = \dot{\eta} = 0$), the final translational equations become

$$\frac{R_{0}}{g_{0}}\ddot{\rho} = \frac{\zeta(2+\zeta)}{(1+\rho)} + \frac{\rho}{(1+\rho)^{2}} + \frac{F_{R}}{g_{0}m}$$
(18)

$$\zeta = \frac{(1 + \rho_{i})\zeta_{i} + \rho_{i} - \rho}{(1 + \rho)} + \frac{1}{\sqrt{\frac{R_{o}}{g_{o}}}(1 + \rho)} \int_{0}^{t} (1 + \rho) \frac{F_{\lambda}}{g_{o}m} dt$$
 (19)

The moment equations written about body axes are

$$\dot{p} = qr \left(\frac{I_Y - I_Z}{I_X} \right) + \frac{M_X}{I_X}$$
 (20)

$$\dot{q} = pr \left(\frac{I_Z - I_X}{I_Y} \right) + \frac{M_Y}{I_Y}$$
 (21)

$$\dot{\mathbf{r}} = pq \left(\frac{\mathbf{I}_{\mathbf{X}} - \mathbf{I}_{\mathbf{Y}}}{\mathbf{I}_{\mathbf{Z}}} \right) + \frac{\mathbf{M}_{\mathbf{Z}}}{\mathbf{I}_{\mathbf{Z}}} \tag{22}$$

In addition, the following auxiliary equations were used:

$$\begin{split} \mathbf{M}_{\mathbf{X}} &= \frac{\partial \mathbf{M}_{\mathbf{X}}}{\partial \delta_{\mathbf{X}}} \, \delta_{\mathbf{X}} \\ \mathbf{M}_{\mathbf{Y}} &= \frac{\partial \mathbf{M}_{\mathbf{Y}}}{\partial \delta_{\mathbf{Y}}} \, \delta_{\mathbf{Y}} \\ \mathbf{M}_{\mathbf{Z}} &= \frac{\partial \mathbf{M}_{\mathbf{Z}}}{\partial \delta_{\mathbf{Z}}} \, \delta_{\mathbf{Z}} \\ \mathbf{m} &= \mathbf{m}_{\mathbf{i}} + \int_{\mathbf{0}}^{\mathbf{t}} \dot{\mathbf{m}} \, d\mathbf{t} \\ \dot{\dot{\mathbf{m}}} &= -\frac{\mathbf{F}}{\mathbf{g}_{\mathbf{e}} \mathbf{I}_{\mathbf{s} \mathbf{p}}} \\ \mathbf{h} &= \mathbf{h}_{\mathbf{0}} + \mathbf{R}_{\mathbf{0}} \rho \\ \dot{\lambda} &= \sqrt{\frac{\mathbf{g}_{\mathbf{0}}}{\mathbf{R}_{\mathbf{0}}}} \frac{1 + \zeta}{1 + \rho} \\ \mathbf{V}_{\lambda} &= \sqrt{\mathbf{R}_{\mathbf{0}} \mathbf{g}_{\mathbf{0}}} (1 + \zeta) \\ \mathbf{V}_{\lambda} &= \sqrt{\mathbf{R}_{\mathbf{0}} \mathbf{g}_{\mathbf{0}}} (1 + \zeta) \\ \mathbf{F} &= \frac{\partial \mathbf{F}}{\partial \delta_{\mathbf{F}}} \, \delta_{\mathbf{F}} \\ \mathbf{F}_{\mathbf{R}} &= \ell_{\mathbf{1}} \mathbf{F} \\ \mathbf{F}_{\lambda} &= \ell_{\mathbf{2}} \mathbf{F} \\ \mathbf{p}_{\mathbf{E}} &= \mathbf{p} - \ell_{\mathbf{3}} \dot{\lambda} \\ \mathbf{q}_{\mathbf{E}} &= \mathbf{q} - \mathbf{m}_{\mathbf{3}} \dot{\lambda} \\ \mathbf{r}_{\mathbf{E}} &= \mathbf{r} - \mathbf{n}_{\mathbf{3}} \dot{\lambda} \\ \mathbf{r}_{\mathbf{E}} &= \mathbf{r} - \mathbf{n}_{\mathbf{3}} \dot{\lambda} \end{split}$$

$$l_{1} = 2(a^{2} + s^{2}) - 1$$

$$l_{2} = 2(ab + cs)$$

$$l_{3} = 2(ac - bs)$$

$$m_{1} = 2(ab - cs)$$

$$m_{2} = 2(b^{2} + s^{2}) - 1$$

$$m_{3} = 2(as + bc)$$

$$n_{1} = 2(ac + bs)$$

$$n_{2} = 2(bc - as)$$

$$n_{3} = 2(c^{2} + s^{2}) - 1$$

$$\dot{s} = -\frac{1}{2}(ap_{E} + bq_{E} + cr_{E}) + kds$$

$$\dot{a} = \frac{1}{2}(sp_{E} - cq_{E} + br_{E}) + kda$$

$$\dot{b} = \frac{1}{2}(cp_{E} + sq_{E} - ar_{E}) + kdb$$

$$\dot{c} = \frac{1}{2}(-bp_{E} + aq_{E} + sr_{E}) + kdc$$

where

$$d = 1 - (s^2 + a^2 + b^2 + c^2)$$

and k is a gain factor determined empirically on the computer.

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- Barker, L. Keith; and Sparrow, Gene W.: An Analytical Investigation of a Simplified Thrust-Vector Orientation Technique for Establishing Lunar Orbits. NASA TN D-3296, 1966.
- 2. Mechtly, E. A.: The International System of Units Physical Constants and Conversion Factors. NASA SP-7012, 1964.

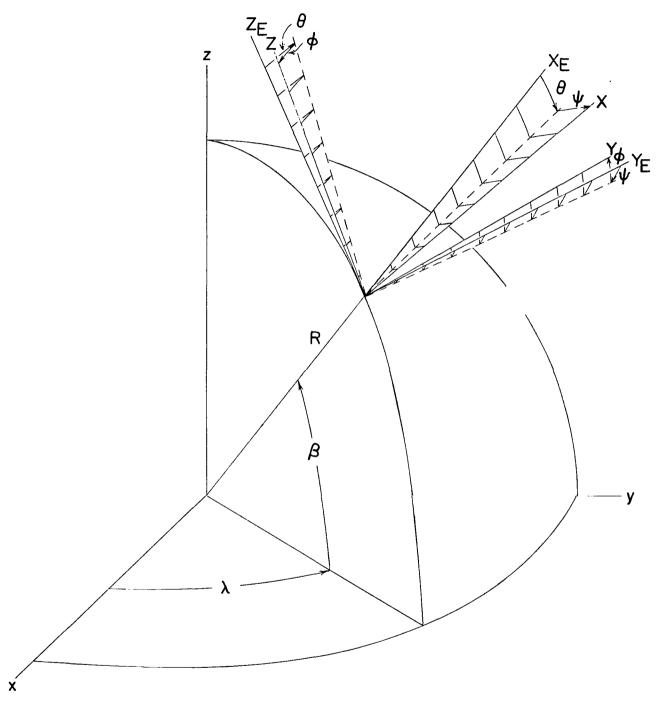


Figure 1.- Assumed axis systems.

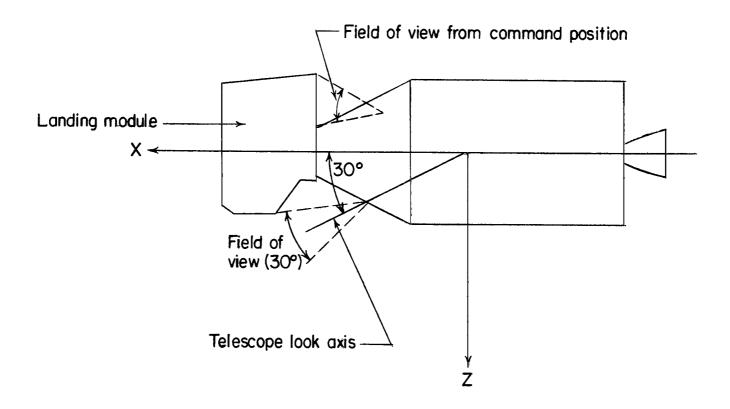
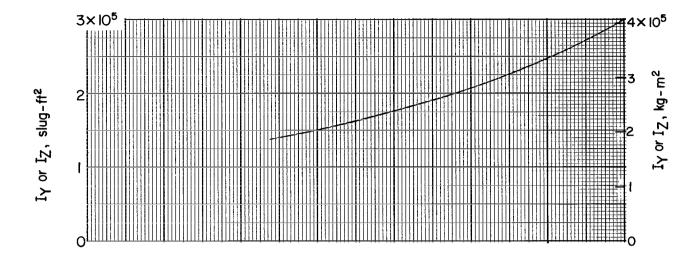


Figure 2.- Vehicle configuration assumed for investigation.



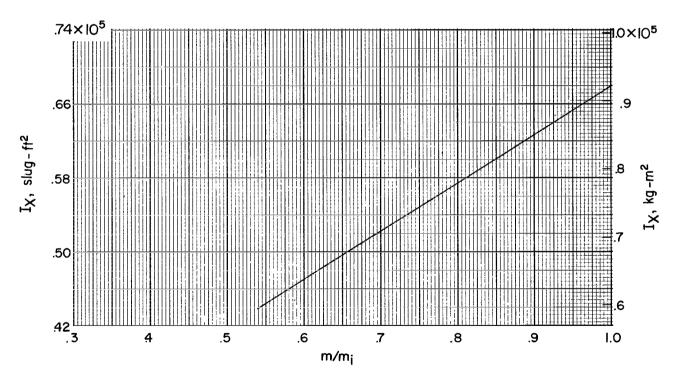
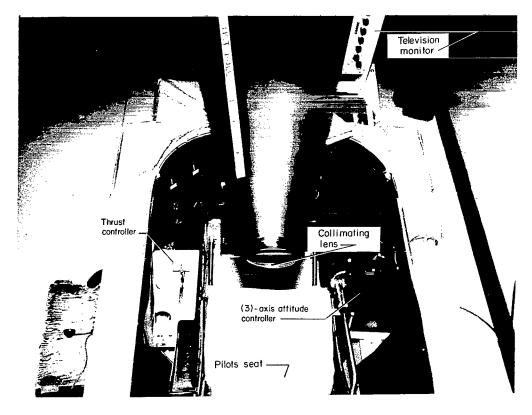


Figure 3.- Variation of vehicle moments of inertia with vehicle mass.



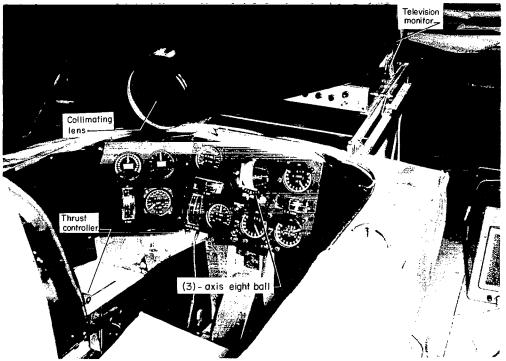


Figure 4.- General layout of cockpit.

L-66-1102

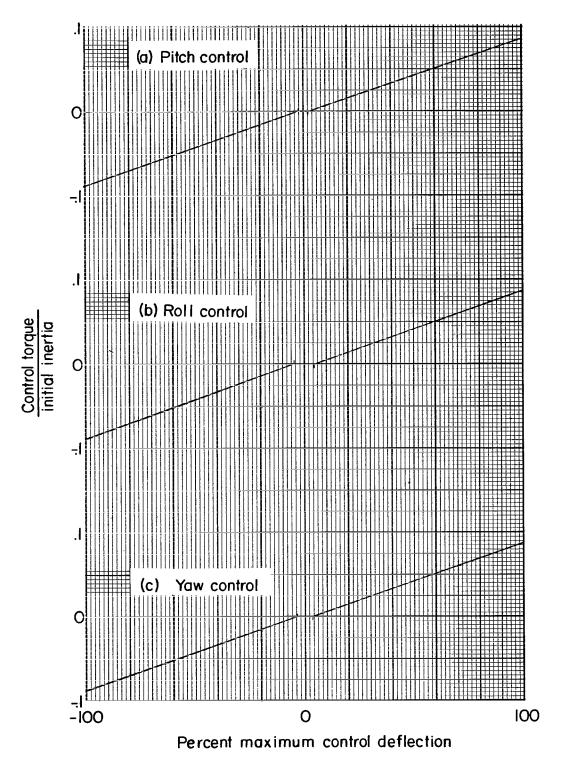


Figure 5.- Initial variation of control torque with control deflection.

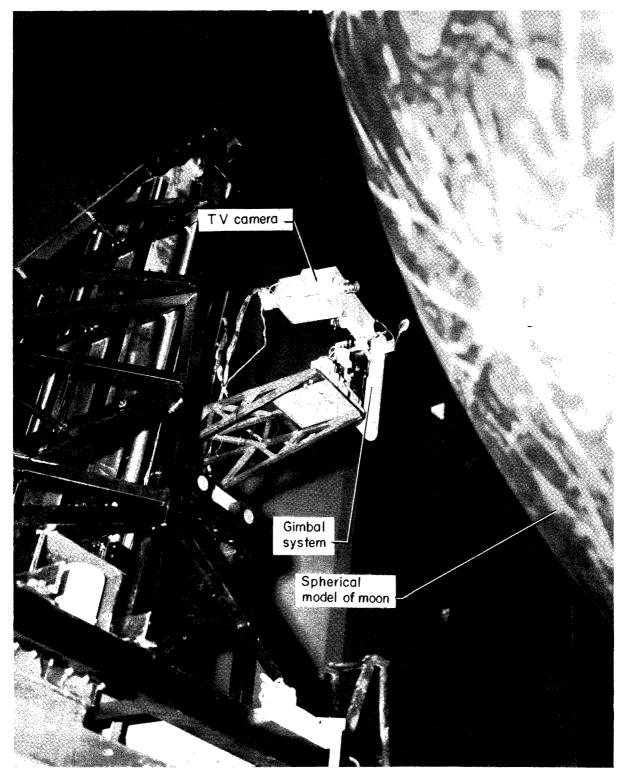


Figure 6.- Image generation equipment.

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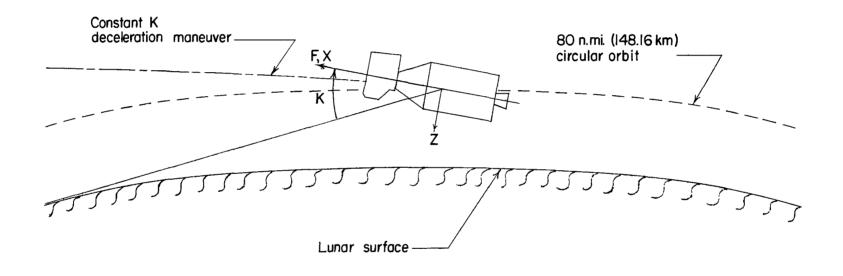
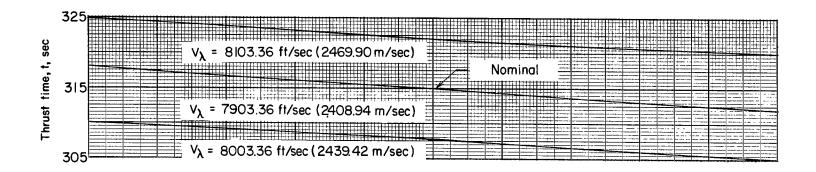


Figure 7.- Lunar orbit injection maneuver.



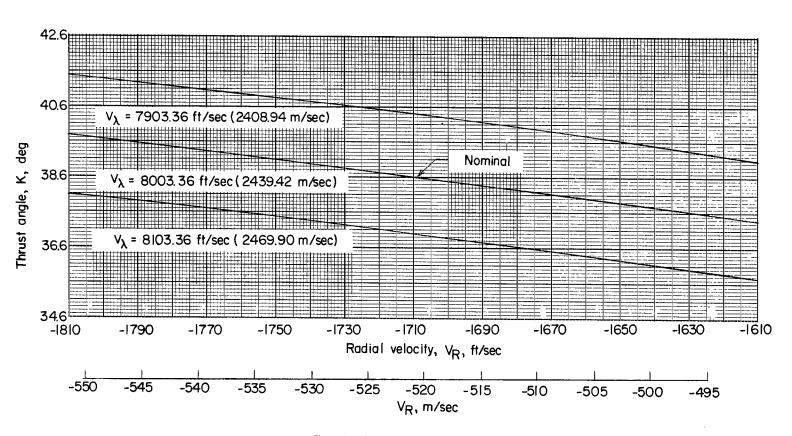


Figure 8.- Correction curves for initial velocity errors.

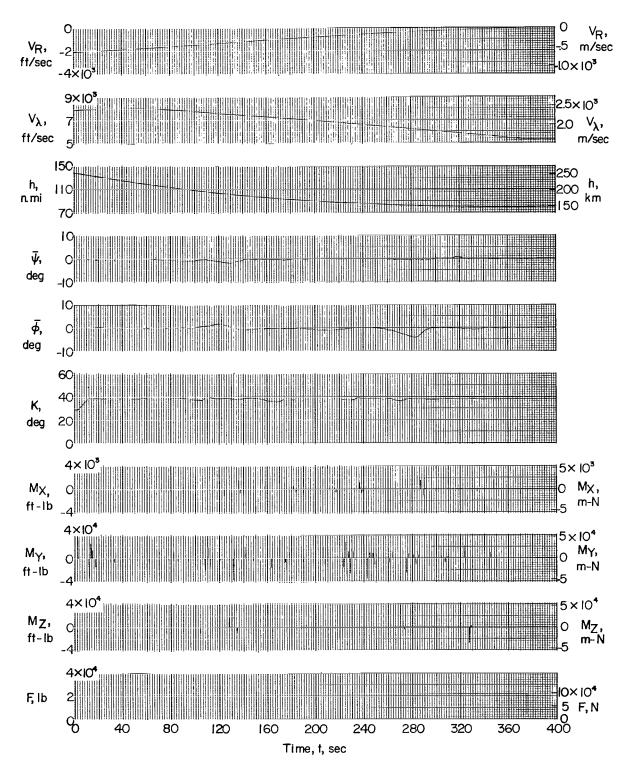
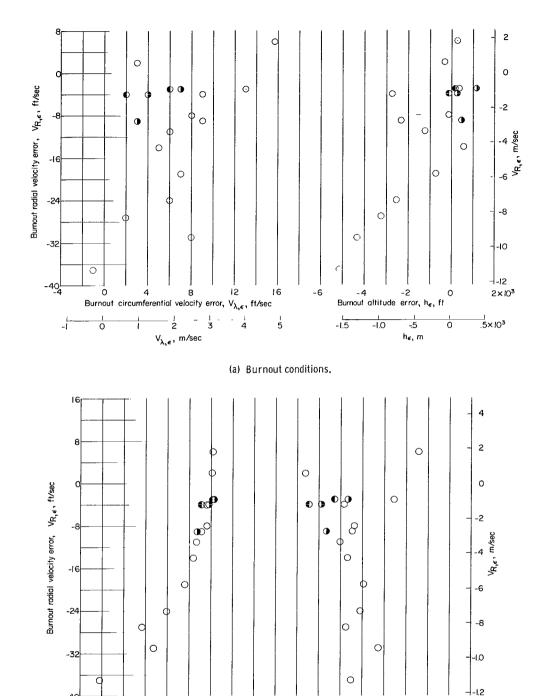


Figure 9.- Typical flight history of nominal trajectory.



(b) Resulting orbit characteristics.

∟ Apocynthion altitude, h_q, n.mi

ha, km

Figure 10.- Summary of results of nominal flights. (Shading used to distinguish between different data for flights having same burnout radial velocity error.)

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-40^L

ـــ Pericynthion altitude, hp, n. mi

h_p, km

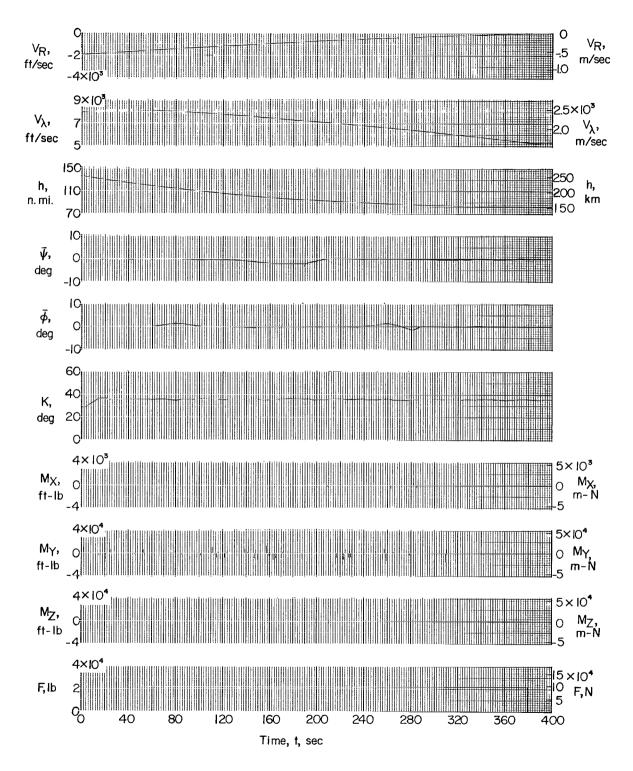


Figure 11.- Typical flight history of off-nominal trajectory.

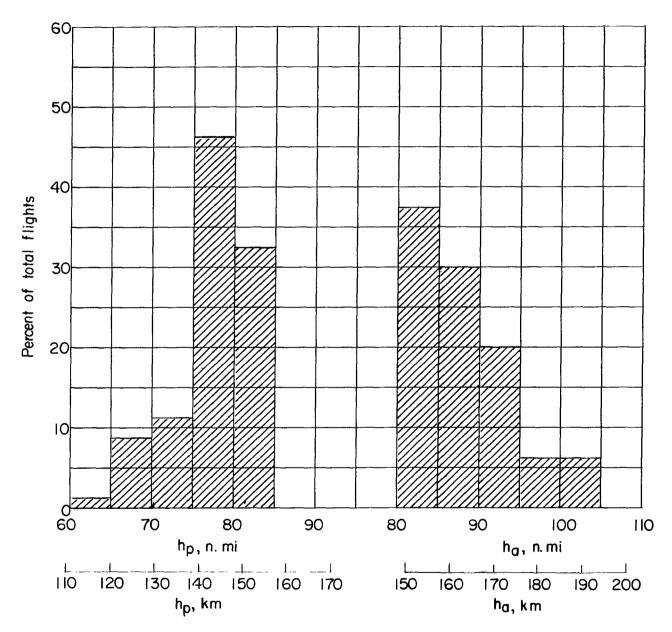


Figure 12.- Pericynthion and apocynthion altitudes of orbits established from nominal and off-nominal trajectories.

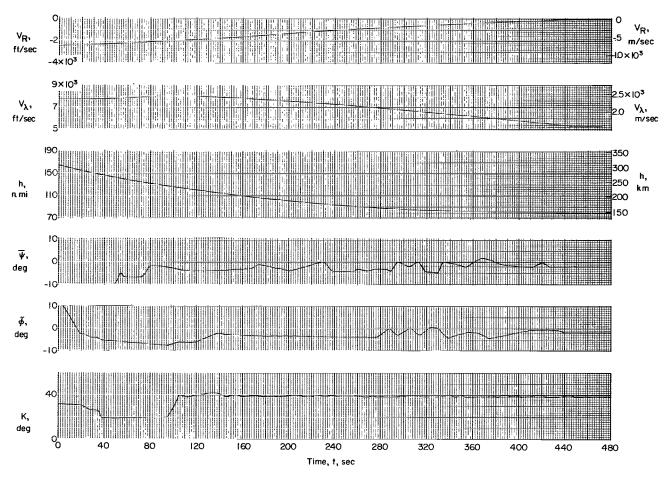


Figure 13.- Typical flight history of nominal trajectory using no instrumentation.

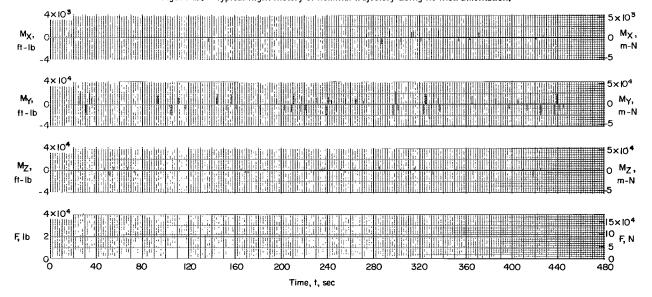


Figure 13.- Concluded.

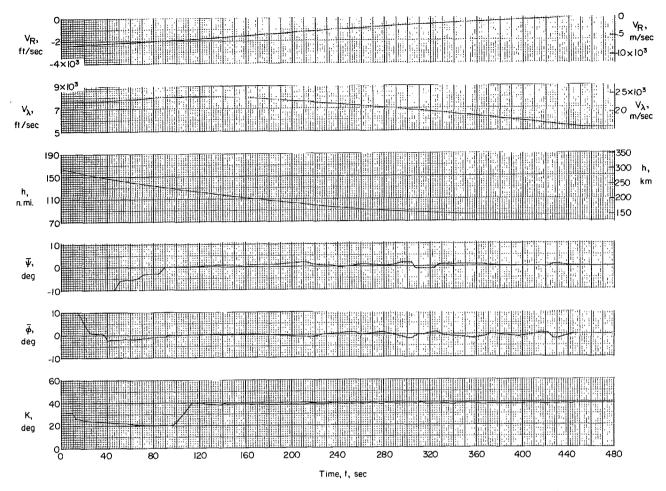


Figure 14.- Typical flight history of nominal trajectory using instrumented attitude rate information.

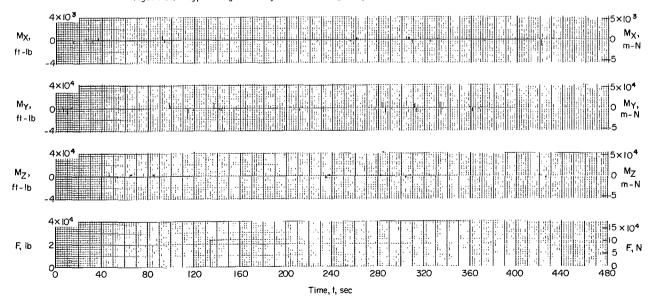


Figure 14.- Concluded.

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